2201. Even functions have graphs with $x = 0$ as a line of symmetry; odd functions have graphs with $(0, 0)$ as a centre of rotational symmetry. Show that this function has a graph which is neither.

2202. (a) This is false; give a counterexample.
(b) This is true; use the fact that kites have a line of symmetry.

2203. The quadratic expression can be factorised, and then the factor theorem can be used.

2204. Sketching graphs can help. Consider cases with different numbers of roots in the solution set S.

2205. Draw a force diagram. Then, consider vertical equilibrium and moments around e.g. the lower support. Solve to find both reaction forces.

2206. Solve to find the intersections of the curves in terms of k. Then set up an equation involving a single definite integral (of the y difference between the curves). Solve for k.

2207. Square the equations, add them, and then simplify using the first Pythagorean trig identity.

2208. This is another way of expressing a first-principles derivative. Rather than letting a small difference h tend to zero, we take two x values p and q, and let them both tend to zero. The difference between them is given, then, by $p - q$, which will tend to zero. As usual, the technique is to engineer this factor on the top, so that it will cancel with the bottom. Then the limit can safely be taken.

2209. Divide this up into cases: start with all the black beads together, then with two of the black beads together, then with only one.

2210. This can be done by the chain rule. However, it is much simpler to use a Pythagorean trig identity.

2211. Square both sides of the equation, but note that, in doing so, you introduce extra points to the curve. Sketch the squared version, and then restrict it to positive y.

2212. Call the short side of the rectangles x and the long side $\frac{4}{x}$. Then draw a sketch and find the area of overlap in terms of x. Set to $\sqrt{2}$ and solve.

2213. The instruction "make b the subject" is exactly the same instruction as "solve for b". Here, do the latter.

2214. The indefinite integral is a standard one, using the reverse chain rule. Then, use the exact values of the tan function, with reference to either the unit circle or a graph.

2215. (a) To calculate the average velocity, integrate to find the displacement and divide by the time taken.
(b) To find the average speed, you need to find the time at which the velocity becomes negative, and integrate either side of it.

2216. Note that the gradients of the lines are reciprocals. This means that the lines are reflections in a line of the form $y = x + c$.

2217. Each of the circles has unit radius. Their centres, then, are located at four points on the unit circle.

2218. The four sloped faces are symmetrical, while the base is different. So, consider separately the cases in which the base is blue or yellow.

2219. Set up an equation using the binomial distribution formula, writing the binomial coefficients in terms of factorials. Then solve.

2220. This is not correct. Friction does act to oppose motion or potential motion. But the two surfaces in potential motion here are not the car and the road. Consider the bottom of the wheels.

2221. Consider the symmetry of a quadratic graph. In particular, determine the transformation between the graphs of $y = \mathrm{f}(p - x)$ and $y = \mathrm{f}(p + x)$.

2222. Express a as $e^{\ln a}$, simplify using index laws, and then differentiate using the chain rule.

2223. In each case, the question is whether or not the denominator has a root.

2224. Draw a clear diagram, drawing the chord and radii to all relevant points. Then set up an equation for the area of the annulus as the difference between circle areas. You'll get a value for $R^2 - r^2$. Using Pythagoras, you can convert this into a value for the (half) chord.

2225. Build term by term, starting with x^2.

2226. Unwrap the curved surface of the cone to form a sector. This sector has radius l, which is the slant height of the cone.

2227. (a) Hypotheses are claims about the population; the sample is what is then used to test those claims.

(b) Both more than and fewer than the expected number of reactions could lead to rejection of the null hypothesis.

(c) Use $\mu = np$.

(d) The p-value is the probability, assuming the null hypothesis, that a sample as extreme as this one could arise. Compare the p-value to the significance level.

2228. Consider the two integrals as areas on your sketch, which fit together like jigsaw pieces to produce a rectangle.

2229. This is about quadratic symmetry. A function can only be inverted over a domain on which it is one-to-one. Since the domains either side of $x = k$ are one-to-one, this specifies the location of $x = k$ on the parabola.

2230. You can calculate the side length of the hexagon by using Pythagoras on one of the faces of the cube. Then split the hexagon into six equilateral triangles: find its area with $6 \times \frac{1}{2}ab\sin C$.

2231. The "distance between" a line and a curve means the shortest distance between them. This is always along a perpendicular to the curve, which, in this case, is an extended radius. Find the centre and radius of the circle by completing the square, and then use standard circle geometry.

2232. "Write b in terms of a" is the same as saying "Solve for b". This is a quadratic in $\sqrt{b}$.

2233. Since the equation is true whatever the value of a and b, the function $|\text{f}(x)|$ must be identical to the function $\text{f}(x)$.

2234. The first case is a circle. The latter cases are scaled versions of the same, in which y has been replaced by ky. This is equivalent to a stretch factor $\frac{1}{k}$ in the y direction. The case $k = 0$ can be thought of as a "stretch" with infinite scale factor.

2235. (a) Consider the two objects as one.

(b) Set up another NII, and eliminate a.

2236. The volume is given by the cross-sectional area multiplied by the length. The area can be found with the formula $\frac{1}{2}ab\sin C$.

2237. Set up the usual formula for the variance, for the combined sample. Then use $\bar{z} = 0$ to simplify. Split the sum up into its x and y terms: these can be re-expressed as the individual variances.

2238. Using log rules, write the both sides as single logs over base 3. Then take 3 to the power of both sides. Solve the cubic using a polynomial solver, or by spotting a root, or by using a numerical method.

2239. Translate each piece of information into a single equation, and solve simultaneously.

2240. Either use polynomial long division or polynomial "short division", which is simplify taking the factor out term by term.

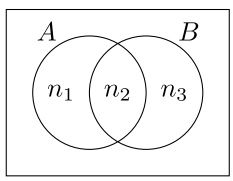
2241. Rearrange each equation to make the trig function the subject. Then square the equations and add them, using the first Pythagorean trig identity.

2242. Draw the bell curve. The normal probabilities are given by areas underneath. Sketch the relevant rectangles and compare areas.

2243. Proceed in the usual fashion, despite the algebraic points: calculate the gradient using $\frac{\text{rise}}{\text{run}}$, and use $y - y_1 = m(x - x_1)$.

2244. The bricks that will initially topple are those on the middle storey. Draw a force diagram for e.g. the right-hand of them, and show that it has a resultant moment clockwise.

2245. This is the first case of the *inclusion-exclusion principle*. A Venn diagram will help. Label the numbers of elements as below:



2246. Enact the differential operator using the product rule, then rearrange to make $\frac{dy}{dx}$ the subject.

2247. There are two ways to do this: either rearrange to $x = y - 1$, substitute and multiply out, or find a cubic in $(x + 1)$ coefficient by coefficient.

2248. Sketch the target, and calculate the areas A_i of the rings, and A of the target itself. Probabilities are then given by A_i/A, and the expectation by the formula $\mathbb{E}(x) = \sum x_i\,\mathbb{P}(X = x_i)$.

2249. The latter graph can be thought of as the result of two output transformations. Sketch this, and then consider the single transformation that has the same effect.

2250. Working with the first line, find $\frac{dy}{dx}$ by the product rule. Then sub your derivative into the LHS of the second line, and simplify to get the RHS.

2251. Expand the harmonic-form expression $R\cos(\theta-\alpha)$ using the relevant compound-angle formula. Then set this as identical to $\sqrt{3}\cos\theta+\sin\theta$ and equate coefficients of $\cos\theta$ and $\sin\theta$. Solve the relevant equations for R and α.

2252. Use the quotient rule, then set the first derivative (and so numerator thereof) to zero.

2253. This is a bit unusual. But you can write the whole thing in terms of $|x|$, so that it can be factorised normally. The key fact is that $x^2 \equiv |x|^2$.

2254. No calculation is needed. However, if you can't see the result, then explicit calculation will do it!

2255. The odd and even cases are different.

2256. Assume that neither of the first two is a multiple of three, and prove that the third one must be.

2257. Rewrite each term over base 2, using the fact that $\log_a b \equiv \log_{a^n} b^n$, and then combine the terms with a log rule.

2258. Remember that correlation is closeness to a *linear* relationship, not merely a relationship.

2259. (a) Differentiate and set the gradient to 2. Solve this cubic and establish the x coordinate at which the tangent occurs.
(b) Find the second derivative.
(c) Find the other intersection point, and then set up a single definite integral of the y distance between the curves.

2260. Use the formula for the sum of the interior angles, which, in radians, is $(n-2)\pi$.

2261. This is a quadratic in $\sqrt{x}$: use the formula.

2262. Write the sum out longhand, i.e. term by term. Then work out the first term and common ratios of the constituent geometric series.

2263. Consider the definition of a point of inflection: the second derivative is zero and changes sign.

2264. (a) This is a perpendicular bisector.
(b) Use Pythagoras.
(c) Solve simultaneously.

2265. Consider the LHS and RHS separately, each as an expression. Simplify and show that they are the same. The simplification is best done by taking out common factors before expanding.

2266. Draw a force diagram for one of the vertices of the triangle of string.

2267. Differentiate implicitly, using the chain rule.

2268. Carry out both integrals, combining the $+c_1$ and $+c_2$ into a single $+c$ on the RHS. Then substitute the initial conditions to find c.

2269. Find the mean of the interior angles of a pentagon first.

2270. Set the first derivative to zero to find stationary points. Hence, find the equation of the dashed line $y=k$. Then set up a single definite integral with integrand $\frac{1}{10}x^4-x^2-k$.

2271. This is a chain of the type that gives the chain rule its name: $\mathrm{f}(\mathrm{g}(\mathrm{h}(x)))$. Differentiate the outside function first, then differentiate the inside function by a new instance of the chain rule.

2272. Call the output of the function y. Then rearrange to make x the subject.

2273. Draw a clear sketch, with the circle tangent to the rhombus at four points. Name half of one of the interior angles α, and find the side lengths in terms of it. Then use a double-angle formula.

2274. (a) Carry out the expansion, and you should see the numbers 362 and 209 appear.
(b) Work out what you would need to equate the result in (a) to, in order to reach the desired approximation. Then explain, by considering the magnitude of $\sqrt{3}-2$, why such an equation is approximately true.

2275. Consider whether there are any x values at which $\sec x$ and $\cot x$ are zero.

2276. There are ${}^7\mathrm{C}_4$ outcomes. Find out how many of them have no two shaded regions sharing a border.

2277. You could set up a general point $\left(p,\sqrt{1-p^2}\right)$, and find the general equation of a tangent. It's much easier, however, to use the fact that the curve is a semicircle.

2278. Substitute the info given into the formulae for the mean and variance, and solve a quadratic in n.

2279. Show that the curve $y = \mathrm{f}(x)$ can has exactly one stationary point. Then argue that there can be at most one x intercept either side of it.

2280. Set up Pythagoras's theorem and differentiate it with respect to time.

2281. (a) The range is the set of outputs reachable from the given domain. So, what are the possible values for the standard deviation of a sample?

(b) An invertible function must be one-to-one. Does every sample in the domain have a unique standard deviation? In other words, can you construct two such samples which have the same standard deviation?

2282. Use the product and chain rules to find the first and second derivatives. Show that, at some x, the first and second derivatives are zero, and that the second derivative changes sign.

2283. Draw a force diagram, deciding on the direction of friction, and resolve parallel and perpendicular to the slope. Find the magnitude of the friction with the perpendicular equation, and then the tension with the parallel one.

2284. Use the reverse chain rule, and the fact that the integral of $\frac{1}{x}$ is $\ln|x|$. Don't forget the mod signs; you'll need them.

2285. Find $\frac{dx}{du}$, then reciprocate to find $\frac{du}{dx}$. Substitute into the LHS as an expression and simplify to zero.

2286. Differentiate for first and second derivatives. Find and classify the SPs.

——— Alternative Method ———

Factorise to $y = (x-1)^2(x+1)^2$, and consider the multiplicity of the roots.

2287. This formula calculates probabilities conditioned on A from probabilities conditioned on B. Hence, draw a tree diagram conditioned on B and consider $\mathbb{P}(B \mid A)$.

2288. (a) λ can be thought of as a linear scale running along AB.

(b) The key fact is that $(c-a)\mathbf{i} + (d-b)\mathbf{j}$ is the vector $\overrightarrow{AB}$.

2289. Exponentiate both sides, and then simplify using log rules to find a parabola. Be careful to check the validity of all regions of the resulting curve: using a log rule can introduce new solutions.

2290. Use the angle in a semicircle theorem.

2291. Use $\cos 2x = 2\cos^2 x - 1$, rearranged to make $\cos^2 x$ the subject. Then use the reverse chain rule.

2292. Use $\mathbb{P}(2 \text{ H} \mid 1 \text{ or } 2 \text{ H}) = \dfrac{\mathbb{P}(2 \text{ H})}{\mathbb{P}(1 \text{ or } 2 \text{ H})}$.

2293. These are a hyperbola and a circle. Substitute for y and show that the resulting equation has double roots.

2294. Use the second Pythagorean trig identity to find a quadratic in $\tan x$. Factorise to solve.

2295. Find the derivative $\frac{dy}{dx}$, and then substitute to get an identity in x. Equate coefficients.

2296. Take moments around O.

2297. Multiply out using the binomial expansion, noting that half of the terms are going to cancel. Simplify the rest and solve. Make sure to check the validity of any roots you find.

2298. This is the region within a circle, and above a mod graph.

2299. Use the compound-angle formulae.

2300. The subtraction of 1 isn't relevant here, since the term "decreasing" concerns only the gradient. Think about (sketch/visualise) the behaviour of $y = x^n$: are there points with negative gradient?

——— End of 23rd Hundred ———